

Programming Problem

Stable Marriage Using Backtracking

The Problem:

You have n men and n women, and their preference rankings of each other, and you need to match them up so that the total matching is “**stable**.”

The preference rankings:

You are given $2 \times n$ arrays, **mp** (men’s preference) which gives the men’s ranking of the women, and **wp** (women’s preference) which gives the women’s ranking of the men.

So $mp[i][j]$ gives man i ’s ranking of woman j and likewise for the women’s ranking of the men in wp .

For example in the following tables we have $n=3$ and the women and men are “named” 0, 1 or 2 and the ranking are in the range 0 = highest, 1 second highest and 2 lowest.

```
int mp[3][3]={0,2,1,
               0,2,1,
               1,2,0};
```

```
int wp[3][3]={2,1,0,
               0,1,2,
               2,0,1};
```

So man 1 assigns woman 2 the rank of 1 (i.e. second highest) and prefers woman 0 the best.

What is a stable matching?

A matching is stable if there are no “**instabilities**.” Say the match assigns m_1 to w_1 and m_2 to w_2 . An instability is the situation where there is a mutual greater preference for the other person’s partner than for one’s own. For example m_1 would prefer w_2 to w_1 and likewise w_2 prefers m_1 to m_2 .

How to do it:

Use the same approach that we used for the one dimensional eight queens problem. In the array q , $q[i]$ is the woman assigned to man i .

The main program stays the same, besides the limits on the loops. All that needs to change is the `ok` function. It could look something like this:

```
bool ok(int q[], int col) {
```

```
    col indicates the new man and q[col] the new woman proposed to be added to the match.
```

We need to do 2 tests:

1. If the new woman has already been assigned to some man then return false
 2. Check the new pair (new man, new woman) against **each** existing pair consisting of (current man, current woman) to see if the new pair would make the match unstable. So
 - a. if the current man prefers the new woman to his partner and
 - b. if the new woman prefers the current man to her partner
 - i. this is unstable, so return false
 - c. if the new man prefers the current woman to his partner and
 - d. if the current woman prefers the new man to her partner
 - i. This is unstable, so return false
 - e. if there are no instabilities introduced with any of the existing (i.e. “current”) couples, so we return true.
- ```
}
```

**Example:** Here are the preference tables MP and WP from before.

|   | MP |   |   |   | WP |   |   |
|---|----|---|---|---|----|---|---|
|   | 0  | 1 | 2 |   | 0  | 1 | 2 |
| 0 | 0  | 2 | 1 | 0 | 2  | 1 | 0 |
| 1 | 0  | 2 | 1 | 1 | 0  | 1 | 2 |
| 2 | 1  | 2 | 0 | 2 | 2  | 0 | 1 |

Will “2 0 1” be a solution? This is the following match:

| Man | Woman |
|-----|-------|
| 0   | 2     |
| 1   | 0     |
| 2   | 1     |

What about “2 1 0”?

**Input Data:**

For this program use the arrays mp and wp above. The data will thus be “given” and not obtained by reading it in.

**Output:**

Print out all stable matchings, one per line. This is the same output that we did with the one dimensional eight queens program.